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# Tachyonic inflation in a warped string background

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Abstract: We analyze observational constraints on the parameter space of tachyonic inflation with a gaussian potential and discuss some predictions of this scenario. As was shown by Kofman and Linde, it is extremely problematic to achieve the required range of parameters in conventional string compactifications. We investigate if the situation can be improved in more general compactifications with a warped metric and a varying dilaton. The simplest examples are the warped throat geometries that arise in the vicinity of of a large number of space-filling D-branes. We find that the parameter range for inflation can be accommodated in the background of D6-branes wrapping a three-cycle in type-IIA. We comment on the requirements that have to be met in order to realize this scenario in an explicit string compactification.

Keywords: Superstrings and Heterotic Strings, Tachyon Condensation, Cosmology of Theories beyond the SM.

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Contents



## 1. Introduction

Inflation [1, 2] is an attractive idea that explains the homogeneity and isotropy of the universe as well as the observed spectrum of density perturbations. Observations of the cosmic microwave background [3, 4] increasingly constrain the class of viable models of inflation. Constructing such models in string theory is therefore an important challenge. See [5] for a review of string-inspired inflation models.

In string theory, the open string tachyon is a natural candidate to play the role of the inflaton. The tachyonic instability is related to the presence of an unstable D-brane in the theory. Brane decay as a time-dependent process was first considered by Sen [6] and and the possibilities for driving cosmological inflation were explored by many authors [7]–[11]. An important objection to the tachyonic inflation scenario was made by Kofman and Linde [12]. They showed that it is extremely difficult to accommodate realistic inflation in conventional string compactifications. The problem is, roughly speaking, that there is no natural small parameter to suppress the energy scale of inflation; hence inflation occurs at Planck energies and the density perturbations produced during the inflationary stage are much too large.

In this note, we will explore whether more general string backgrounds can be found where an additional small parameter is present and where the problems for tachyonic inflation can be overcome. A natural generalization which can be realized in string theory is that of a warped compactification, where the 4-dimensional metric contains an overall factor that can vary of the compactified space. Since the parameters governing the tachyonic action depend also on the dilaton, we achieve more freedom by allowing it to vary over the compact manifold as well. This is a natural generalization of the definition of warping as a varying dilaton contributes to the warp factor in the Einstein frame.

We consider the simplest examples of warped backgrounds, which can be obtained by wrapping a large number of space-filling D-branes on a cycle of the compact manifold. The backreaction then produces a 'throat region' with significant warping and varying dilaton. We find that the parameter range for inflation can be accommodated in the background of D6-branes wrapping a three-cycle in type-IIA. What makes inflation possible in this background is the property that the string coupling decreases faster than the (string frame) warp factor as we approach the branes. In order to trust the supergravity approximation far enough into the warped region, the number of D6-branes has to be large: at least of order $10^6$ if we want to achieve slow-roll and of order $10^{13}$ if we insist on a realistic perturbation spectrum. This poses the problem of finding a way to cancel the RR tadpoles without interfering with inflation. We speculate on how this might be achieved, leaving a more detailed investigation for the future.

This paper is organized as follows. Section 2 gives an overview of slow-roll and density perturbations in tachyonic inflation. Section 3 analyzes the parameter constraints and predictions of tachyonic inflation with a gaussian potential. In section 4 we review the objections raised by Kofman and Linde, and in section 5 we derive a condition for improving the situation in a warped background. In section 6 we consider warped brane backgrounds and find that inflation can be improved in the D6-brane background which we analyze in more detail in section 7. In section 8 we discuss requirements that have to be met in order to realize the scenario in an explicit string compactification.

## 2. Overview of tachyonic inflation

In this section we will briefly review the properties of tachyonic inflation. Our discussion will closely follow [10]. The premise is that the effective dynamics of the universe during the inflationary stage is described by 3+1 dimensional gravity coupled to a scalar field $T$ with an action of the Born-Infeld type:

$$S = \int d^4x\sqrt{-g}\left(\frac{{M_{\rm pl}}^2}{2}R - AV(T)\sqrt{1+B\partial_\mu T\partial^\mu T}\right). \tag{2.1}$$

Here, $V(T)$ is a positive definite potential with a maximum at $T=0$ and normalized to $V(0)=1$; $A$ and $B$ represent positive constants. As we will see later, if the model arises from a string compactification in the presence of an unstable D-brane, $A$ and $B$ turn out to depend on the string length and, importantly, on the dilaton and the warp factor.

A homogeneous tachyon field $T(t)$ acts as a perfect fluid source for gravity with energy density and pressure given by:

$$\rho = \frac{AV(T)}{\sqrt{1-B\dot{T}^2}} \tag{2.2}$$

$$p = -AV(T)\sqrt{1-B\dot{T}^2}\,. \tag{2.3}$$

The dynamics follows from the equation of motion for the tachyon and the Friedmann equation:

$$\frac{\ddot{T}}{1-B\dot{T}^2} + 3H\dot{T} + \frac{V'}{BV} = 0 \tag{2.4}$$

$$H^2 = \frac{1}{3{M_{\rm pl}}^2}\frac{AV(T)}{\sqrt{1-B\dot{T}^2}} \tag{2.5}$$

where $H$ is the Hubble parameter. Accelerated expansion occurs when $\rho + 3p < 0$ or, equivalently,

$$\dot{T}^2 < \frac{2}{3B}\,. \tag{2.6}$$

Inflation will persist for many e-folds if the scalar field rolls sufficiently slowly, i.e. if the friction term in (2.4) dominates over the acceleration term:

$$\ddot{T} < 3H\dot{T}\,. \tag{2.7}$$

During slow-roll inflation, the equations of motion can be approximated by

$$3H\dot{T} + \frac{V'}{BV} = 0 \tag{2.8}$$

$$H^2 = \frac{1}{3{M_{\rm pl}}^2}AV\,. \tag{2.9}$$

The replacement of the second-order system (2.4), (2.5) by the first-order equations (2.8), (2.9) is justified as the solution of the latter acts as an attractor [10]. We define slow-roll parameters $\epsilon_1,\ \epsilon_2$ by

$$\epsilon_1 = \frac{3B}{2}\dot{T}^2 \tag{2.10}$$

$$\epsilon_2 = 2\frac{\ddot{T}}{H\dot{T}} \tag{2.11}$$

so that the conditions for slow-roll inflation (2.6), (2.7) become $\epsilon_1 \ll 1,\ |\epsilon_2| \ll 1$. Inflation ends when $\epsilon_1 = 1$. Using the slow-roll equations of motion, we can rewrite these conditions as requirements on the flatness of the potential $V(T)$:

$$\epsilon_1 \simeq \frac{{M_{\rm pl}}^2}{2AB}\frac{V'^2}{V^3} \ll 1 \tag{2.12}$$

$$\epsilon_2 \simeq \frac{{M_{\rm pl}}^2}{AB}\left(-2\frac{V''}{V^2} + 3\frac{V'^2}{V^3}\right) \ll 1\,. \tag{2.13}$$

The dependence of the slow-roll parameters on $A,\ B$ can also be understood by making a field redefinition to a canonically normalized field. For inflation near the top of the potential (which is the case we will study), the canonically normalized field is $\tilde{T} = \sqrt{ABV}T \approx \sqrt{AB}T$. Hence the slow-roll parameters, which each contain two derivatives of $V$ with respect to $\tilde{T}$, are proportional to $(AB)^{-1}$. The fact that the constants $A,\ B$, and hence the dilaton and the warp factor, enter in the slow-roll parameters will be essential for our construction of a string background in which $\epsilon_1,\ \epsilon_2$ are naturally small.



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The number of e-folds between the tachyon value $T$ and the end of inflation $T_e$ is given by

$$N(T) \simeq \frac{AB}{{M_{\rm pl}}^2} \int_{T_e}^{T} \frac{V^2}{V'} dT \,.$$

Various observables related to scalar and gravitational fluctuations were computed in [10]. To first order in the slow-roll parameters, the scalar and gravitational power spectra are given by

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{8\pi^2 {M_{\rm pl}}^2 \epsilon_1} \tag{2.14}$$

$$\mathcal{P}_g(k) = \frac{2H^2}{\pi^2 {M_{\rm pl}}^2} \tag{2.15}$$

where the right hand side is to be evaluated at $aH = k$. To leading order, the tensor-scalar ratio $r$, the scalar spectral index $n$ and the tensor spectral index $n_T$ are given by:

$$r \equiv \frac{\mathcal{P}_g}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon_1 \tag{2.16}$$

$$n - 1 \equiv \frac{d\ln \mathcal{P}_{\mathcal{R}}(k)}{d\ln k} = -2\epsilon_1 - \epsilon_2 \tag{2.17}$$

$$n_T \equiv \frac{d\ln \mathcal{P}_g(k)}{d\ln k} - 2\epsilon_1 \,. \tag{2.18}$$

The running of the spectral indices is, to leading order,

$$\frac{dn}{d\ln k} = -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3 \tag{2.19}$$

$$\frac{dn_T}{d\ln k} = -2\epsilon_1\epsilon_2 \tag{2.20}$$

where

$$\epsilon_2\epsilon_3 = \frac{{M_{\rm pl}}^4}{(AB)^2}\left(2\frac{V'''V'}{V^4} - 10\frac{V''V'^2}{V^5} + 9\frac{V'^4}{V^6}\right).$$

These relations are identical to the ones for a canonical scalar where the standard slow- roll parameters $\epsilon$, $\eta$ (see e.g. [3] for a definition) are related to ours by $\epsilon = \epsilon_1$, $\eta = 2\epsilon_1 - \frac{1}{2}\epsilon_2$. The difference between the tachyon and a canonical scalar shows up at the next order in the slow-roll parameters [10]. Consistency relations between the observables reduce the number of independent ones to 4, which can be taken to be $(\mathcal{P}_{\mathcal{R}}, n, r, dn/d\ln k)$. For a given potential $V(T)$, the observational limits on two observables can be used to constrain $A$ and $B$, giving two predictions for the other observables.

## 3. Tachyonic inflation with a gaussian potential: constraints and predictions

In this section we will analyze the restrictions imposed on tachyonic inflation with a gaussian potential from the requirements of slow-roll inflation and agreement with observations of the cosmic microwave background anisotropy. We perform an analysis to first order in the slow-roll parameters following the standard procedure [13].



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An unstable space-filling D-brane in superstring theory is described by an action of the form (2.1), where we take the potential to be gaussian:

$$V(T) = e^{-T^2}\,.$$

This potential has been argued [14] to give a good description for small $T$ and we will assume that it is sufficiently accurate throughout the whole inflationary stage. Inflation starts when the brane decays and the tachyon rolls down from the top of the potential. This is a model of eternal inflation [15] because of quantum fluctuations occasionally returning the field to the top of the potential.

We will now discuss the constraints on the parameters $A, B$ from observations as well as some predictions of the model. Because of the property $V'' < V'^2/V$, the last 60 or so 'observable' e-folds can occur either at negative curvature $V'' < 0$ or at small positive curvature $0 < V'' < V'^2/V$. We will call these 'region I' and 'region II' respectively.[1] The slow-roll parameters are

$$\begin{aligned}\epsilon_1 &\simeq \frac{2{M_{\rm pl}}^2}{AB} T^2 e^{T^2} \\ \epsilon_2 &\simeq \frac{4{M_{\rm pl}}^2}{AB}(1+T^2)e^{T^2} \\ \epsilon_2\epsilon_3 &\simeq \frac{16{M_{\rm pl}}^4}{(AB)^2} T^2(2+T^2)e^{2T^2}\,.\end{aligned} \tag{3.1}$$

Hence inflation occurs if $T$ is sufficiently small and

$$\frac{AB}{4{M_{\rm pl}}^2} \gg 1\,.1 \tag{3.2}$$

The end of inflation is determined by

$$T_e^2 e^{T_e^2} \simeq \frac{AB}{2{M_{\rm pl}}^2} \tag{3.3}$$

and the number of e-folds until the end of inflation is

$$N(T) \simeq \frac{AB}{4{M_{\rm pl}}^2}\left(E_1(T^2) - E_1(T_e^2)\right) \tag{3.4}$$

where $E_1$ is the exponential integral $E_1(x) = \int_x^\infty du \frac{e^{-u}}{u}$.

To compare with observations, we have to evaluate the observables at the time when cosmological scales cross the horizon. In the numerical estimates below, we will take this to be the value $T_*$ for which $N_* \simeq 60$, even though the latter value depends on the details of reheating, which are uncertain for this model (see section 8). The value of $T_*$ determines whether observable inflation occurs in region I ($T_* < 1$) or region II ($T_* > 1$).

We now discuss the predictions of the model in more detail. We use the observational limits on $n$ and $\mathcal{P}_{\mathcal{R}}$ from [3] to constrain $A$ and $B$ and treat the values of $r$, $dn/d\ln k$

[1]These regimes correspond to models of 'class A' and 'class B' respectively in the classification of [3].

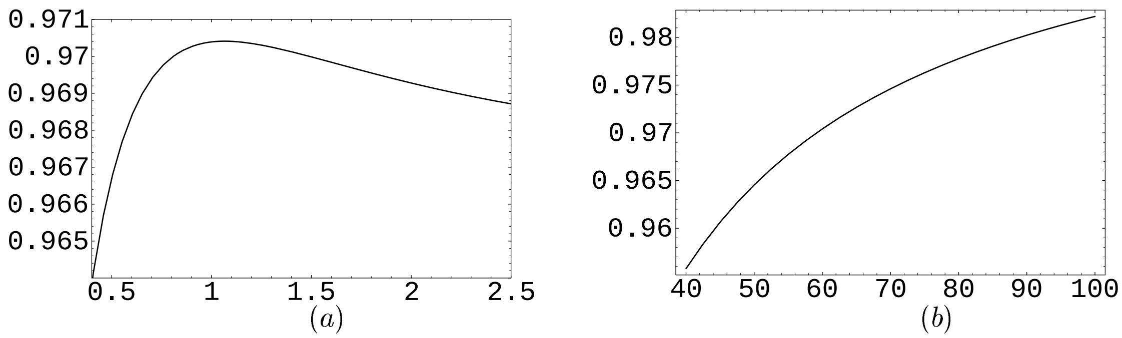


**Figure 1:** $(a)$ $n$ in function of $T_*$ for $N_* = 60$. $(b)$ $n_{\rm max}$ as a function of $N_*$.

as predictions of the model. Concretely, we proceed as follows: for a given value of $n$ we use the relations (3.3), (3.4) to solve for the corresponding values of $T_*$ and $AB/{M_{\rm pl}}^2$. We can then determine $r$, $dn/d\ln k$ from (2.16), (2.19), (3.1) and obtain $A$ through the relation (2.14) and the observational constraint on $\mathcal{P}_{\mathcal{R}}$, leading to

$$\frac{A}{{M_{\rm pl}}^4} \simeq 24\pi^2 \frac{\epsilon_{1*}}{V_*} \times 0.71 \times 2.95 \times 10^{-9}\,.$$

One distinctive feature of the model is that the spectral index $n$ is not a monotonic function of $T_*$ but reaches a maximum at $n_{\rm max} \simeq 0.9704$ for $T_* \simeq 1.06$ (see figure 1$a$). As we vary $N_*$ between 40 and 100, the value of $n_{\rm max}$ varies between $0.956 \le n_{\rm max} \le 0.982$ (see figure 1$b$). Current observations [3] are consistent with $n$ in the range $0.94 \le n \le 1$ so the model predicts a value in the lower part of this range with a minimum deviation from the scale invariant value $n = 1$. This feature is a likely candidate for future observational falsification.

In region I, the allowed range of $n$ corresponds to the following values for $r$:

$$0.0086 \le r \le 0.079\,.$$

Although these values lie within the current observational bound $r \le 0.16$ for this type of model [3], they may be large enough to come within range of future experiments. Figure 2$a$ shows $r$ as a function of $n$ for various values of $N_*$. Linked to this relatively sizeable fraction of gravitational waves is the prediction of a relatively high inflationary energy scale $\rho^{1/4}$:

$$9.8 \times 10^{15}\,\text{Gev} \le \rho^{1/4} \le 1.7 \times 10^{16}\,\text{GeV}\,.$$

Hence inflation occurs quite close to the susy GUT scale in this model.

The running of the spectral index is very small and negative:

$$-1.9 \times 10^{-4} \ge \frac{dn}{d\ln k} \ge -4.9 \times 10^{-4}$$

which is well within the allowed range $-0.02 \le dn/d\ln k \le 0.004$ [3]. Figure 2$b$ shows $dn/d\ln k$ as a function of $n$ for various values of $N_*$.

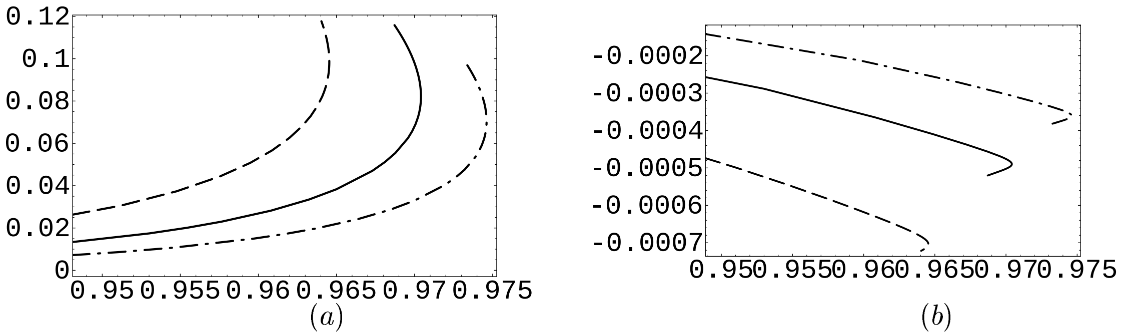


**Figure 2:** ($a$) $r$ versus $n$ and ($b$) $dn/d\ln k$ versus $n$ for $N_* = 50$ (dashed line), $N_* = 60$ (solid line) and $N_* = 70$ (dot-dashed line).

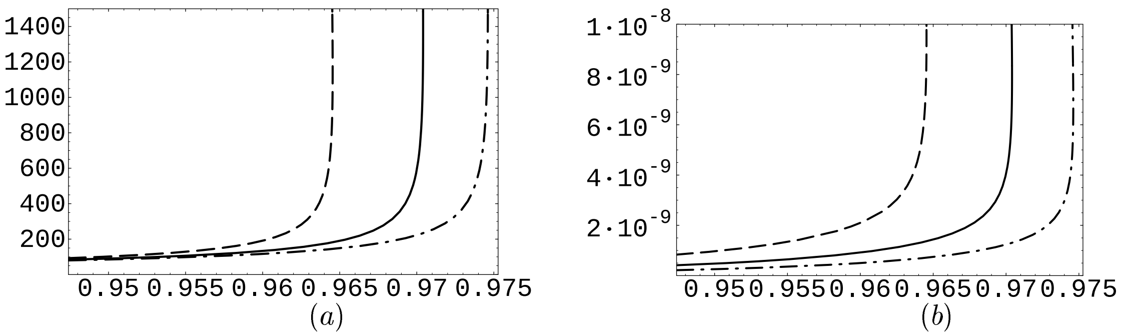


**Figure 3:** ($a$) $AB/{M_{\rm pl}}^2$ versus $n$ and ($b$) $A/{M_{\rm pl}}^4$ versus $n$ for $N_* = 50$ (dashed line), $N_* = 60$ (solid line) and $N_* = 70$ (dot-dashed line).

The combination $AB/{M_{\rm pl}}^2$ ranges over

$$70.4 \leq \frac{AB}{{M_{\rm pl}}^2} \leq 1102$$

and $A/{M_{\rm pl}}^4$ varies between

$$2.7 \times 10^{-10} \leq A \leq 6.7 \times 10^{-9} \,.$$

These constants are plotted as a function of $n$ in figure 3.

The conclusions are similar for inflation in region II. As $T_*$ increases, the ratio $r$ becomes even more sizeable, while $dn/d\ln k$ becomes more negative. For example at $T_* = 2$ they take the values $r \simeq 0.11$ and $dn/d\ln k \simeq -5.1\times10^{-4}$. The energy scale during inflation also increases, e.g. $\rho^{1/4} \simeq 1.9 \times 10^{16}$ at $T_* = 2$. The constants $AB/{M_{\rm pl}}^2$ and $A/{M_{\rm pl}}^4$ grow more rapidly in this regime, e.g. $AB/{M_{\rm pl}}^2 \simeq 6.4 \times 10^4$ and $A/{M_{\rm pl}}^4 \simeq 1.8 \times 10^{-7}$ at $T_* = 2$. The predictions in region II are less reliable as it is not clear whether a gaussian potential gives the correct description for large field values. It has been argued [14] that, as $T \to \infty$, one should instead use an exponential potential which was studied extensively in [10].

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## 4. Problems for tachyonic inflation in unwarped compactifications

In this section, we review a potentially fatal problem for the tachyonic inflation scenario sketched above when arising from a string compactification in the presence of an unstable D-brane. Kofman and Linde showed in [12] that the values of the parameters $A$ and $B$ needed for realistic inflation are far outside the range that can be accomodated by a conventional string compactification. By 'conventional' we mean that the 10-dimensional geometry is a product of 3+1 dimensional spacetime and a 6-dimensional compact manifold and that the dilaton is constant. In such a compactification, the 4-dimensional Planck mass is given by

$$M_{\rm pl}{}^2 = \frac{v}{g^2\alpha'}$$

where

$$v = \frac{2V_6}{(2\pi)^7\alpha'^3} \tag{4.1}$$

with $g$ the closed string coupling and $V_6$ the volume of the compactification manifold. In order for the effective action (2.1) to be applicable, the $\alpha'$ and string loop corrections should be small which requires $g < 1$ and $v > 1$.

For simplicity, let's consider the case of a space-filling non-BPS D3-brane in type-IIA (the conclusions are unchanged if one takes a higher dimensional brane wrapping some of the internal directions). The parameters $A$ and $B$ are then given by

$$A = \frac{\sqrt{2}}{(2\pi)^3 g\alpha'^2} \tag{4.2}$$

$$B = 8\ln 2\alpha' \,. \tag{4.3}$$

The condition (3.2) for slow-roll becomes

$$\frac{g}{v} \gg \frac{(2\pi)^3}{2\sqrt{2}\ln 2} \simeq 127\,. \tag{4.4}$$

So in order to get many e-folds of inflation we need either a large string coupling or small compactification volume.

The situation gets much worse if, in addition, we require the density fluctuations to fall within observational constraints. For example, we saw in the previous section that, for inflation in region I, the parameter $A$ satisfies

$$\frac{A}{M_{\rm pl}{}^4} < 6.7 \times 10^{-9}\,.$$

Combining this with (4.4) implies that $v$ has to be extremely small:

$$v \ll 5.7 \times 10^{-13}\,.$$

The same order of magnitude was found in [12] from constraints on gravitational wave production. For such tiny values of the compactified volume, $\alpha'$ corrections almost certainly render the action (2.1) unreliable.



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## 5. The role of warping

The discussion of the previous section assumed a conventional compactification in which the 10-dimensional geometry is a product of 3+1-dimensional spacetime and a compact 6-dimensional manifold. However, string theory allows more general, warped compactifications [16, 17] where the 10-dimensional string frame metric is of the form

$$ds^2 = e^{2C(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n \,. \tag{5.1}$$

The warp factor $e^{2C}$ can become very small in a certain region while its average is of order 1 [17]. Processes taking place in the warped region are redshifted leading to a hierarchy of energy scales. It is natural to ask whether one can find warped compactifications where the slow-roll parameters are similarly suppressed and the problems sketched in the previous sections can be overcome. A crucial extra ingredient is that we shall also allow the dilaton to vary over the compact manifold:

$$\phi = \phi_0 + \phi(y) \,. \tag{5.2}$$

The 4-dimensional Planck mass in such a compactification is

$${M_{\rm pl}}^2 = \frac{\tilde{v}}{g^2 \alpha'}$$

where $g = e^{\phi_0}$ and the 'warped volume' $\tilde{v}$ now depends on the warp factor and the dilaton:

$$\tilde{v} = \frac{2}{(2\pi)^7 \alpha'^3} \int d^6 y \sqrt{g_6} e^{-2\phi + 2C} \,.$$

We will consider compactifications where the average value of $e^{-2\phi+2C}$ is of order 1 so for our purposes there is no difference between $\tilde{v}$ and $v$ defined in (4.1). Embedding a non-BPS D3-brane in this background[2] leads to an effective action of the form (2.1) with the expressions (4.2), (4.3) for $A$ and $B$ now replaced by

$$A = \frac{\sqrt{2} e^{4C-\phi}}{(2\pi)^3 g \alpha'^2} \tag{5.3}$$

$$B = 8 \ln 2 \alpha' e^{-2C} \,, \tag{5.4}$$

while the condition (4.4) for slow-roll inflation gets replaced by

$$\frac{g e^{2C-\phi}}{\tilde{v}} \gg 127 \,. \tag{5.5}$$

Hence we see that slow-roll can be facilitated in backgrounds where locally

$$e^{2C-\phi} \gg 1 \,. \tag{5.6}$$

[2]For the type-IIB case one should use a $D3 - \bar{D}3$ pair, which is also described by an action of the form (2.1) but with a complex scalar $T$.

## 6. Warped backgrounds from space-filling D-branes

Simple examples of warped backgrounds are the ones obtained by wrapping a large number of space-filling D-branes on a cycle of the compact manifold. The backreaction then produces a 'throat region' with significant warping and varying dilaton. Let us consider a large number $N$ of Dp-branes ($3 \leq p \leq 6$) wrapping a $p-3$ cycle. Such backgrounds are a generalization of the $p = 3$ brane case considered by Verlinde in [18]. The explicit supergravity solution can easily be written down for a toroidal compactification; the modifications for the more general case are unimportant for our purposes. Using coordinates $(x^\mu, y^a, y^i)$, $\mu = 0, \ldots, 3$, $a = 1, \ldots, p-3$, $i = p-2, \ldots, 9-p$, the string frame geometry in the vicinity of the branes can be approximated by

$$ds^2 = H_p^{-1/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dy^a dy^a\right) + H_p^{1/2} dy^i dy^i \tag{6.1}$$

$$e^{2\phi} = g^2 H_p^{\frac{3-p}{2}} \tag{6.2}$$

with

$$H_p = \left(\frac{r_p}{r}\right)^{7-p} \tag{6.3}$$

$$(r_p)^{7-p} = 2^{5-p}\pi^{\frac{5-p}{2}}\Gamma\left(\frac{7-p}{2}\right) gN(\alpha')^{\frac{7-p}{2}} \tag{6.4}$$

and we have defined a radial coordinate $r^2 = y^i y^i$. We have also assumed that $r/r_p \ll 1$. For large $r$, the geometry is modified and glues smoothly into a compact manifold. The $\alpha'$ corrections to the supergravity approximation (6.2) are unimportant provided that $gN \gg 1$ and, for $p > 3$, [19]

$$\frac{r_p}{r} \ll (gN)^{\frac{4}{(p-3)(7-p)}} \tag{6.5}$$

where, on the right hand side, we have neglected a numerical factor greater than one. String loop corrections can be safely ignored in the region of interest ($r_p/r \gg 1$). For the moment we shall assume that the number of branes $N$ can be made arbitrarily large and come back to the restrictions from RR tadpole cancellation in the discussion in section 8.

These geometries are of the warped type (5.1), (5.2) with

$$e^{2C-\phi} = \left(\frac{r_p}{r}\right)^{\frac{(7-p)(p-5)}{4}}.$$

Since $r_p/r \gg 1$ we see that the condition (5.6) for improving slow-roll inflation can be met only for $p = 6$ while for the other values of $p$ the warping makes matters worse!

## 7. Improved inflation in the background of D6-branes

Hence we shall work from now on with the $D6$-brane background in type-IIA with a non-BPS $D3$-brane embedded in the throat region. The parameters $A$ and $B$ in Planck units are then

$$\frac{A}{{M_{\rm pl}}^4} = \frac{\sqrt{2}}{(2\pi)^3}\frac{g^3}{\tilde{v}^2}\left(\frac{r_p}{r}\right)^{-1/4} \tag{7.1}$$

$$B{M_{\rm pl}}^2 = 8\ln 2\frac{\tilde{v}}{g^2}\left(\frac{r_p}{r}\right)^{1/2}. \tag{7.2}$$

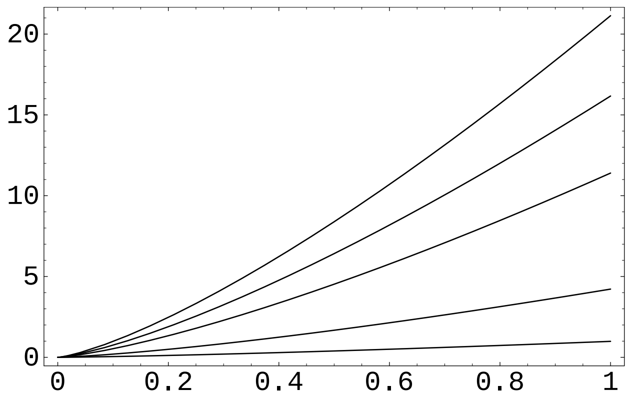


**Figure 4:** $\tilde{v}$ versus $g$ for inflation with, from top to bottom, $n = 0.94$, $n = 0.95$, $n = 0.96$, $n = 0.97$ (region I) and $n = 0.97$ (region II).

Eliminating $r_p/r$ we find a relation between $\tilde{v}$ and $g$:

$$\tilde{v} \simeq 0.056 \left(\frac{A}{M_{\rm pl}{}^4}\right)^{-2/3} (BM_{\rm pl}{}^2)^{-1/3} g^{4/3} . \tag{7.3}$$

Using this relation we can explore which values of $\tilde{v}$ and $g$ can give rise to realistic inflation by plugging in the appropriate values for $A$ and $B$ from the analysis in section 3. From figure 4 we see that it is now possible to accommodate realistic inflation with $\tilde{v} > 1$ and $g < 1$, the situation becoming better for inflation in region I for smaller values of $n$. For example, for $n = 0.97$ in region I, we need $g > 0.34$ in order to have $\tilde{v} > 1$, while for $n = 0.94$ we only need $g > 0.10$. The situation becomes worse for inflation in region II where, for example for $n = 0.97$, one needs $g > 1$ to obtain $v > 1$.

We can also estimate the the number of D6-branes required in order in order to be able trust the supergravity approximation (6.2) in the region of significant warping. This number turns out to be quite large. The condition (6.5) gives the minimal number of D6-branes $N_{\rm min}$ as

$$N_{\rm min} = \frac{1}{g}\left(\frac{r_p}{r}\right)^{3/4} .$$

Let us first look at the number of branes required to meet the slow-roll condition (5.5), which can be written as

$$\frac{g^{4/3}}{v} \gg \frac{127}{N_{\rm min}^{1/3}} .$$

Hence we need the number of branes to be at least of order $10^6$ or so to accommodate slow-roll in the $g < 1$, $\tilde{v} > 1$ region. Requiring that, in addition, the perturbation spectrum falls within observational constraints further increases $N_{\rm min}$. Using (7.1), (7.2) we can express $N_{\rm min}$ in terms of $A$ and $B$:

$$N_{\rm min} \simeq 5.88AB^2 .$$

Using the values of $A$, $B$ appropriate for inflation in region I, one finds that one needs $N_{\rm min}$ to be at least of order $10^{14}$.

## 8. Discussion

We saw in section 3 that a tachyonic scalar with a gaussian potential can provide a viable inflationary scenario within current observational bounds. The main predictions are an upper limit on the scalar spectral index $n \leq 0.98$ and a sizeable production of gravitational waves with the scale of inflation close to the susy GUT scale. We proposed a mechanism to attain the required parameter range in string theory by embedding the unstable brane in the throat geometry produced by a large number of D6-branes wrapping a three-cycle in the compact manifold. We shall now comment on some of the hurdles that need to be overcome in order to realize this idea in a concrete string compactification.

First of all, our setup requires a compactification that includes a sufficient number of D6-branes. Since the D6-branes carry RR charge, consistency requires that the RR tadpoles be cancelled by introducing objects with negative RR charge. In a supersymmetric Minkowski space compactification, this requires introducing a sufficient number of orientifold O6-planes [20]. Although it is not unthinkable that there exist orientifold compactifications with of the order $10^6$ D6-branes required for getting e-folds,[3] it is doubtful whether the number of branes can be as high as the order $10^{14}$ needed for obtaining realistic density perturbations. Another way of cancelling the tadpoles is by introducing anti-D6 branes. In a Minkowski space compactification this breaks supersymmetry and jeopardizes the long-term stability of the compactification due to the attractive forces between branes and anti-branes; the question is then whether it is possible to find a setup that is sufficiently stable to allow the inflationary phase to take place. Yet another possibility would be to start with a compactification to 4-dimensional anti-de Sitter space before introducing supersymmetry-breaking effects such as the unstable D3-brane. In anti-de Sitter compactifications it is possible to cancel the RR tadpoles of an arbitrary number of D6-branes with anti-branes in a stable manner without breaking supersymmetry.[4] Explicit examples were constructed in [22]. Adding the unstable brane (and possibly other supersymmetry breaking effects to raise the value of the cosmological constant) to such a background may yield a configuration that is sufficiently stable to support inflation. We leave these issues for further investigation.

We also want to stress that, in the present work, we have implicitly assumed that it is possible to stabilize the scalar moduli of the compactification. Much progress has been made recently in constructing flux compactifications in type-IIB/F-theory with all moduli stabilized [23, 24], and the interplay between moduli stabilization and various inflationary scenarios has been addressed [25]. In our case, it is of particular importance to stabilize those moduli that are sourced by the tachyon, such as the volume modulus of the compactified manifold, in order for inflation to work. The issue of moduli stabilization is also important for obtaining a sensible late-time cosmology, as the value of the potential for the moduli contributes to the cosmological constant.

In the light of this comment one might wonder whether tachyonic inflation could also be realized in other corners of the string theory moduli space, where the problem of mod-

[3]For example, in the case of F-theory compactifications, examples are known which allow up to order $10^4$ D3-branes [21].

[4]We would like to thank Frederik Denef for pointing this out to us.



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uli stabilization is under better control. In the present example, the condition (5.5) for improving slow-roll was met because, in the D6-brane background, the string coupling $e^{\phi}$ decreases faster than the the warp factor $e^{2C}$ when we approach the branes. Alternatively, one might look for more complicated warped backgrounds which have, for example, a region where $e^{2C}$ is of order one while $e^{\phi}$ becomes very small.

Finally, a realistic inflation model has to incorporate a mechanism for reheating the universe by converting the energy contained in the inflaton field into radiation. The proposal of [11], in which unstable branes decay into stable D-branes containing the standard model, is likely to be amenable to the present context.

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